

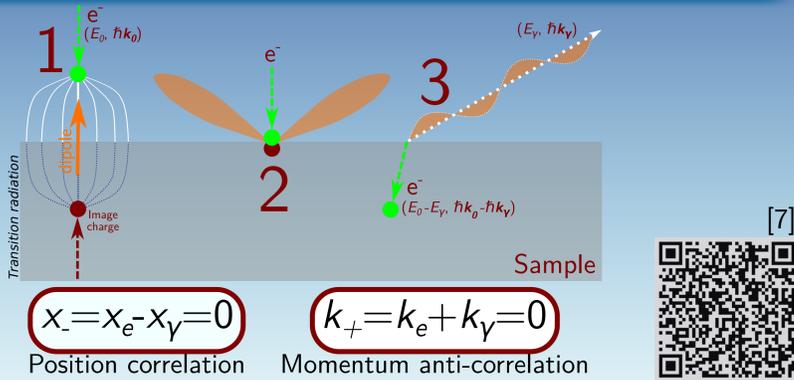
## Quantum Electron Optics

We have great freedom and precision in shaping free electron beams using an electron microscope. Such instrumentation brings enormous possibilities in coupling free electrons to matter or light waves on the nanoscale, revealing pathways for studies of quantum phenomena occurring between electrons and a huge variety of particles and quasi-particles [1,2]. This makes the electron microscope a powerful tool for extending quantum optics experiments to reaches far beyond the limits of traditional photon-based techniques. Quantum entanglement [3,4] is a phenomenon which lies at the heart of quantum science and technology, but until recently, evidence of quantum entanglement involving free electrons has remained elusive [5,6].

In our demonstration of quantum entanglement [5], we quantitatively probe the correlations between electrons and the coherent transition radiation photons that they emit upon interaction with a thin membrane sample. To do this, we adapt coincidence counting and ghost imaging from quantum optics to study the electron-photon pair relationship in both position and momentum spaces.

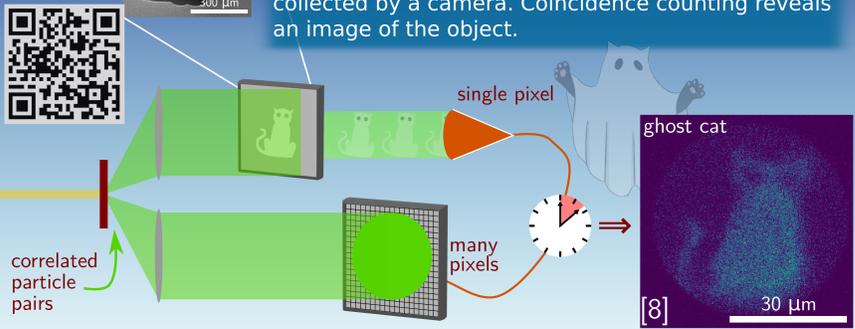
## Coherent Cathodoluminescence

Coherent CL has a predictable phase relationship with the electron beam's evanescent fields. Examples include transition radiation (TR), Cherenkov radiation (CR), diffraction radiation, etc. During a coherent emission event, the sample undergoes no changes and energy/momentum is exchanged only between the electron and photon pair, resulting in energy and (transverse) momentum conservation. Additionally, the photon is emitted from the location of the electron at the time of emission, resulting in strong position correlations.



## Ghost Imaging

In quantum ghost imaging, the correlations between two particles are used to take an image of an object using a particle that never interacted with that object. One particle passes through the object and is collected by a bucket detector. The second particle is collected by a camera. Coincidence counting reveals an image of the object.



## Entanglement bound

To detect the presence of entanglement, we use an entanglement witness: a quantitative criterion to distinguish between separable and non-separable (i.e. entangled) states. Separable states are those where the states of two particles can be described independently, while entangled states cannot.

Both electron and photon must independently obey Heisenberg's uncertainty principle.

$$\Delta x_e \Delta k_e \geq 1/4$$

$$\Delta x_\gamma \Delta k_\gamma \geq 1/4$$

Only if electron and photon states are non-separable can they be simultaneously perfectly correlated in position and anti-correlated in momentum. The states are entangled if the joint uncertainties obey:

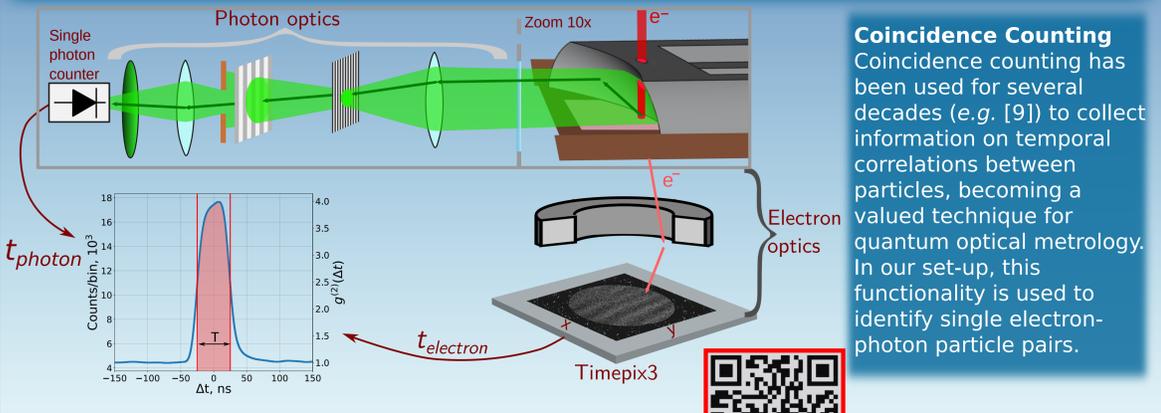
$$\Delta x_-^2 \Delta k_+^2 < 1$$

## References

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## Experimental Details

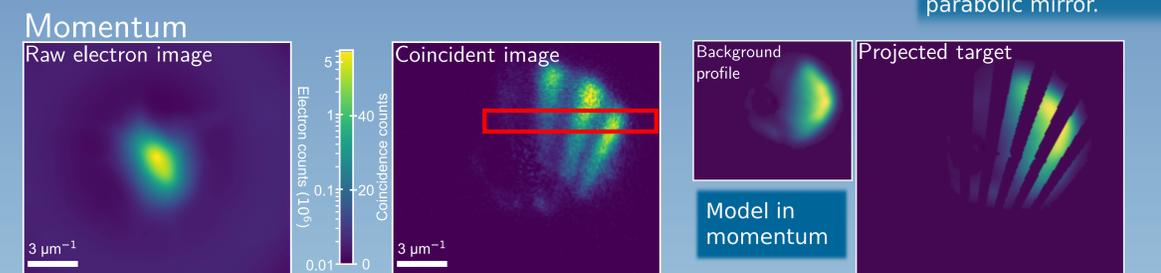
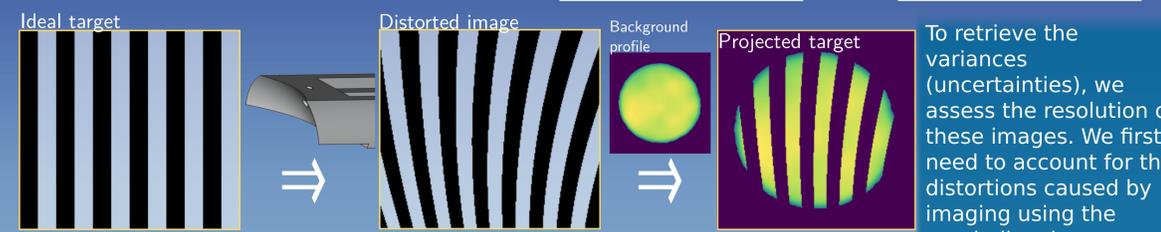
We use a fast Timepix3 (TP3) camera (Advascope) mounted in the spectrometer of an FEI Tecnai TEM, operated at 200 keV, to obtain time-resolved readout of individual electrons. The electron beam is directed through a 50 nm thick silicon membrane. We collect photons using a custom free-space CL holder with a parabolic mirror, which directs the photons through an optical port in the microscope column. The photon optics path contains a moveable calibration target which acts as our object for ghost imaging. It is placed in either the position or momentum plane. After the target, photons are collected by a single photon counting module (SPCM) which can provide photon arrival timestamps. We then use the electron and photon timestamps, synchronized through a Time Tagger (Swabian Instruments), to filter coincident electron-photon pairs.



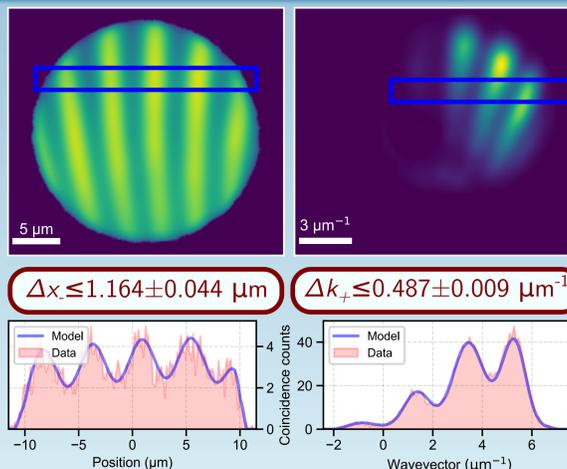
**Coincidence Counting**  
Coincidence counting has been used for several decades (e.g. [9]) to collect information on temporal correlations between particles, becoming a valued technique for quantum optical metrology. In our set-up, this functionality is used to identify single electron-photon particle pairs.

## Experimental Entanglement Witness

With the target in the photon image plane, we acquire coincidence images of the electron beam image plane. Likewise, with the grating in the momentum plane, we acquire coincidence images of the electron beam momentum plane. After coincidence filtering, the image of the grating lines is revealed, representing the joint electron-photon pair state.

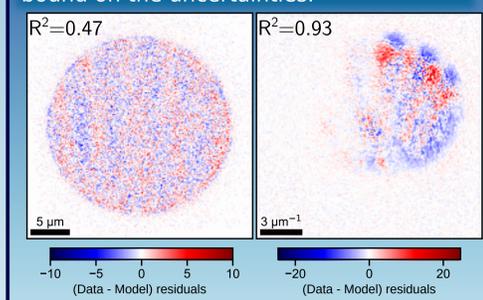


We assume a Gaussian point spread function is responsible for the imperfect imaging and fit our data using a 2D model of these 'ideal' lines convolved with a Gaussian.



## Residuals

Residuals of the fit show some remaining features in the pattern, including patterns originating in the Poissonian nature of the counting statistics. Despite these imperfections, the fit provides an upper bound on the uncertainties.



$$\Delta x_-^2 \Delta k_+^2 \leq 0.321 \pm 0.027 < 1$$

The final joint uncertainty product is lower than the entanglement bound, thus demonstrating entanglement in position and momentum between electron-photon pairs.

## Mixed Basis

As further confirmation, we measured the ghost images in mixed bases of the joint state. That is, with the lines in the momentum plane ( $k_y$ ), we measured the electron in the image plane ( $x_e$ ); and with the lines in the image plane ( $x_\gamma$ ), we measured the electron in the momentum plane ( $k_e$ ). The featureless beam verifies independence of the chosen bases, holding promise for the outcome of more robust entanglement certification schemes [10].

